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Computing the modal characteristics of structures considering soil-structure interaction effects

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Abstract

Modal analysis of structures is usually performed based on finite element models where the structures are considered undamped and fixed at their base, disregarding any interaction with the soil. In some cases though, these modeling assumptions may lead to erroneous estimates. This paper presents a finite element-perfectly matched layers model which is used to compute the modal characteristics of a frame structure considering dynamic soil-structure interaction effects. The modal characteristics of the structure are computed with respect to the soil stiffness revealing the key effects of dynamic soil-structure interaction. The analysis shows how soil-structure interaction affects the fundamental as well higher modes of the structure, involving increase in modal damping ratios and engendering complex valued mode shapes.

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Keywords: dynamic soil-structure interaction; perfectly matched layers; modal analysis; quadratic eigenvalue problem; non-proportional damping

1. Introduction

Modal analysis is an essential part of the design process of many civil engineering structures as it allows for the estimation of their modal characteristics (eigenfrequencies, modal damping ratios and mode shapes), the knowledge of which is crucial in numerous applications. Typical examples arise in the fields of earthquake and bridge engineering where the computation of modal characteristics is of prime importance in order to avoid or mitigate possible structural resonance effects due to earthquake or traffic excitation, respectively. These effects can affect the structural integrity or reduce the serviceability of buildings and bridges.

In civil engineering, modal analysis is usually performed based on finite element models, where structures are commonly considered undamped and fixed at their base, disregarding any interaction with the soil. Although these modeling assumptions yield acceptable results for many problems, they will lead to erroneous estimates for the modal characteristics in some cases.

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Research on dynamic soil-structure interaction in earthquake engineering shows that this interaction generally increases the flexibility of a structure and its effective damping due to the energy loss throughout the soil [1]. For some types of structures, these interaction effects are non-neglectable and should be accounted for by using appropriate computational models, such as coupled finite element-boundary element or finite element-perfectly matched layers models [2]. In these models, the structure is no longer fixed at its base and the influence of the soil is explicitly taken into account. The computation of the modal characteristics of these more sophisticated models generally requires the solution of computationally challenging transcendental eigenvalue problems. As eigensolvers for this kind of eigenvalue problems are not widely available, the influence of the soil on the modal characteristics of structures is usually treated in a simplified manner or even disregarded.

This paper uses a finite element-perfectly matched layers model to compute the modal characteristics of structures considering dynamic soil-structure interaction effects. The influence of these effects is demonstrated for a two-dimensional structure by investigating the sensitivity of its modal characteristics with respect to the subsoil properties.

2. Methodology

Application of the finite element method for a linear visco-elastic structure that is fixed at its base, gives the following eigenproblem for the computation of its modal characteristics:

$$\left[\boldsymbol{K} + \lambda_m \boldsymbol{C} + \lambda_m^2 \boldsymbol{M}\right] \boldsymbol{\psi}_m = \boldsymbol{0} \tag{1}$$

where $\boldsymbol{K} \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$, $\boldsymbol{C} \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ and $\boldsymbol{M} \in \mathbb{R}^{\mathrm{n}\times\mathrm{n}}$ are the stiffness, damping and mass matrix, $\lambda_m \in \mathbb{C}$ and $\boldsymbol{\psi}_m \in \mathbb{C}^{\mathrm{n}\times 1}$ ($m = 1, ..., 2\mathrm{n}$) are the m-th eigenvalue and m-th eigenvector, and n the number of degrees of freedom in the model. For subcritical damping, the eigensolutions of equation (1) occur in complex conjugate pairs. Furthermore, the real part of the eigenvalues is always negative for stable systems as this ensures that energy can only be dissipated and is not created within the system. The eigenfrequencies $\omega_{\mathrm{r}m} \in \mathbb{R}$ and modal damping ratios $\xi_m \in \mathbb{R}$ of the structure are computed from its eigenvalues as:

$$\omega_{\mathrm{r}m} = |\lambda_m| \quad \text{and} \quad \xi_m = -\frac{\mathrm{Re}(\lambda_m)}{|\lambda_m|} \tag{2}$$

If the structure is proportionally damped, its mode shapes $\boldsymbol{\psi}_m$ are real and all their elements have a relative phase difference of 0° or 180°. In the case of general damping, though, the mode shapes $\boldsymbol{\psi}_m$ are complex valued and the modal displacement ψ_{km} of each degree of freedom k is determined by its modulus $|\psi_{km}|$ and phase $\theta_{km} = \arg(\psi_{km})$ [3]. The overall phase coherence between the n elements of a mode shape $\boldsymbol{\psi}_m$ can be quantified using the modal collinearity factor (MCF) [3] defined as (figure 1):

$$\mathrm{MCF} = 1 - \frac{A_{\mathrm{p}}}{A_{\mathrm{c}}} \tag{3}$$

where A_{c} is the circular area in the complex plane that is defined by the element of $\boldsymbol{\psi}_m$ with the largest magnitude and A_{p} is the convex area in the complex plane that enfolds all elements of $\boldsymbol{\psi}_m$. The MCF takes values from 0 to 1 with values closer to 1 indicating virtually real modes.

When dynamic soil-structure interaction is taken into account, the computation of the modal characteristics of the coupled soil-structure system generally requires the solution of the following eigenproblem:

$$\left[\boldsymbol{K} + \lambda_m \boldsymbol{C} + \lambda_m^2 \boldsymbol{M} + \boldsymbol{K}_{\mathrm{s}}(\lambda_m)\right] \boldsymbol{\psi}_m = \boldsymbol{0} \tag{4}$$

where $\boldsymbol{K}_{\mathrm{s}}(\lambda) \in \mathbb{C}^{\mathrm{n}\times\mathrm{n}}$ represents the dynamic stiffness contribution of the soil, and the number of degrees of freedom n has been enlarged to include any extra degrees of freedom from the soil. In coupled finite element-boundary element models only the additional degrees of freedom of the structure-soil interface need to be included since $\boldsymbol{K}_{\mathrm{s}}(\lambda)$ is given as the dynamic impedance of this interface. Other alternatives for the modeling of $\boldsymbol{K}_{\mathrm{s}}(\lambda)$ include finite element formulations in conjunction with absorbing boundary conditions simulating the semi-infinite extend of the soil. The dynamic stiffness matrix $\boldsymbol{K}_{\mathrm{s}}(\lambda)$ of the soil is not available in a closed functional form and is usually evaluated numerically at a discrete frequencies $i\omega \in \mathbb{I}$. As $\boldsymbol{K}_{\mathrm{s}}(\lambda)$ is generally an intractable non-polynomial function of $\lambda \in \mathbb{C}$, the eigenproblem (4) is more challenging to solve than the quadratic eigenproblem (1).

The dynamic soil-structure interaction problem is formulated in the present work by means of finite elements (FE) and perfectly matched layers (PML) [4]. The structure Ω_b and a limited bounded volume Ω_s of the surrounding soil - referred collectively as the generalized structure $\Omega_\mathrm{r} = \Omega_\mathrm{b} \cup \Omega_\mathrm{s}$ - are modeled with FE, while PML are used to simulate the truncated unbounded soil at the limits of the finite element model (figure 2). In order to artificially attenuate the elastodynamic waves that enter into the PML buffer zone, complex coordinate stretching is applied in Ω_p. For coordinate s, representing the x or y coordinate, the stretched coordinate $\bar{s}$ is defined as:

$$\bar{s} = \int_0^s \lambda_\mathrm{s}(s)\,\mathrm{d}s = s_\mathrm{o} + \int_{s_\mathrm{o}}^{s_\mathrm{t}} \lambda_\mathrm{s}(s)\,\mathrm{d}s \tag{5}$$

where the complex valued stretch function λ_s is defined as:

$$\lambda_\mathrm{s}(s,\lambda) = \alpha_{0\mathrm{s}}(s) + \frac{\alpha_{1\mathrm{s}}(s)}{\lambda} \tag{6}$$

where $\alpha_{0\mathrm{s}}$ and $\alpha_{1\mathrm{s}}$ are polynomial functions that control respectively the profile of attenuation of the evanescent and propagating waves inside the PML buffer zone, and $\lambda \in \mathbb{C}$ is the (complex valued) frequency of analysis.

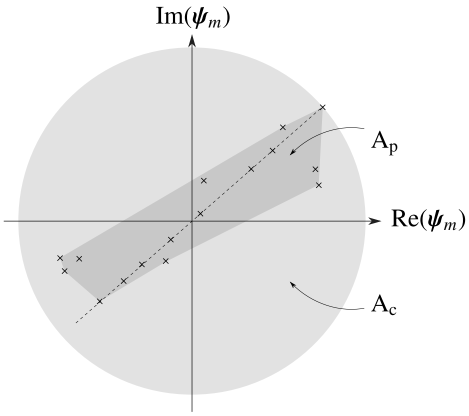

Fig. 1. Definition of MCF in equation (3).

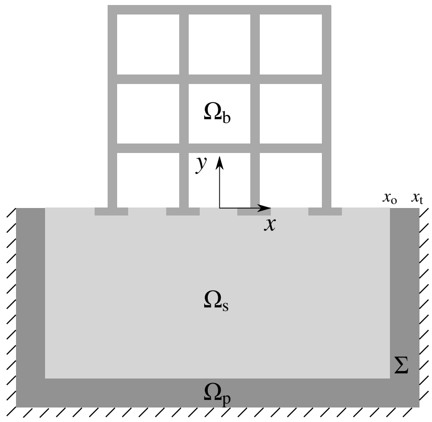

Fig. 2. FE-PML model.

The finite element matrices $\boldsymbol{K}_\mathrm{r} \in \mathbb{R}^{\mathrm{n_r}\times\mathrm{n_r}}$, $\boldsymbol{C}_\mathbf{r} \in \mathbb{R}^{\mathrm{n_r}\times\mathrm{n_r}}$ and $\boldsymbol{M}_\mathrm{r} \in \mathbb{R}^{\mathrm{n_r}\times\mathrm{n_r}}$ of the generalized structure Ω_r are derived following a standard Galerkin displacement based formulation, whereas the dynamic stiffness matrix $\boldsymbol{K}_\mathrm{s}(\lambda) \in \mathbb{C}^{\mathrm{n_s}\times\mathrm{n_s}}$ of the unbounded soil is derived following a mixed formulation where both displacements and stresses are retained as independent variables in Ω_p [4]. For two-dimensional problems, this formulation allows to write $\boldsymbol{K}_\mathrm{s}(\lambda)$ as a quadratic matrix polynomial with respect to $\lambda \in \mathbb{C}$:

$$\boldsymbol{K}_\mathrm{s}(\lambda) = \boldsymbol{K}_{\mathrm{s}0} + \lambda \boldsymbol{K}_{\mathrm{s}1} + \lambda^2 \boldsymbol{K}_{\mathrm{s}2} \tag{7}$$

with $\boldsymbol{K}_{\mathrm{s}0} \in \mathbb{R}^{\mathrm{n_s}\times\mathrm{n_s}}$, $\boldsymbol{K}_{\mathrm{s}1} \in \mathbb{R}^{\mathrm{n_s}\times\mathrm{n_s}}$ and $\boldsymbol{K}_{\mathrm{s}2} \in \mathbb{R}^{\mathrm{n_s}\times\mathrm{n_s}}$. The free vibration equation of the coupled structure-soil system can now be written as:

$$\boldsymbol{T}^\mathrm{T}\left(\begin{bmatrix}\boldsymbol{K}_\mathrm{r} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{K}_{\mathrm{s}0}\end{bmatrix} + \lambda_m \begin{bmatrix}\boldsymbol{C}_\mathrm{r} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{K}_{\mathrm{s}1}\end{bmatrix} + \lambda_m^2 \begin{bmatrix}\boldsymbol{M}_\mathrm{r} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{K}_{\mathrm{s}2}\end{bmatrix}\right)\boldsymbol{T}\begin{bmatrix}\boldsymbol{\psi}_{\mathrm{r}m}\\ \boldsymbol{\psi}_{\mathrm{s}m}\end{bmatrix} = \begin{bmatrix}\mathbf{0}\\ \mathbf{0}\end{bmatrix} \tag{8}$$

where $\boldsymbol{T} \in \mathbb{R}^{(\mathrm{n_r}+\mathrm{n_p})\times\mathrm{n}}$ is a constraint matrix that eliminates the displacement degrees of freedom on the interface Σ from $\boldsymbol{K}_\mathbf{s}$. Obviously, equation (8) corresponds to a quadratic eigenvalue problem of the form:

$$\boldsymbol{Q}(\lambda_m)\,\boldsymbol{\psi}_m = \left[\widetilde{\boldsymbol{K}} + \lambda_m \widetilde{\boldsymbol{C}} + \lambda_m^2 \widetilde{\boldsymbol{M}}\right]\boldsymbol{\psi}_m = \mathbf{0} \tag{9}$$

where $\widetilde{\boldsymbol{K}}$, $\widetilde{\boldsymbol{C}}$ and $\widetilde{\boldsymbol{M}}$ can be interpreted as stiffness-like, damping-like and mass-like matrices, respectively.

The first step to solving the quadratic eigenvalue problem of equation (9) is to transform it into a larger linear matrix pencil which has the same eigenvalues as the original problem. The following linear matrix pencil is used [5]:

$$\boldsymbol{L}(\lambda_m)\,\tilde{\boldsymbol{\psi}}_m = (\boldsymbol{A} - \lambda_m \boldsymbol{B})\,\tilde{\boldsymbol{\psi}}_m = \left(\begin{bmatrix}\widetilde{\boldsymbol{K}} & \mathbf{0}\\ \mathbf{0} & \boldsymbol{I}\end{bmatrix} - \lambda_m \begin{bmatrix}-\widetilde{\boldsymbol{C}} & -\widetilde{\boldsymbol{M}}\\ \boldsymbol{I} & \mathbf{0}\end{bmatrix}\right)\tilde{\boldsymbol{\psi}}_m = \mathbf{0} \tag{10}$$

Next, a standard linear eigensolver can be applied to compute the eigenpairs ($\lambda_m \in \mathbb{C}$, $\tilde{\boldsymbol{\psi}}_m \in \mathbb{C}^{2n\times1}$) of the linear matrix pencil (10). Subsequently, the eigenvectors $\boldsymbol{\psi}_m \in \mathbb{C}^{n\times1}$ of the original problem (9) can be extracted from $\tilde{\boldsymbol{\psi}}_m$ since $\tilde{\boldsymbol{\psi}}_m^{\mathrm{T}} = \{\boldsymbol{\psi}_m^{\mathrm{T}}, \lambda_m\boldsymbol{\psi}_m^{\mathrm{T}}\} \in \mathbb{C}^{2n\times1}$.

The computed eigenpairs ($\lambda_m \in \mathbb{C}$, $\boldsymbol{\psi}_m \in \mathbb{C}^{n\times1}$) include both physical modes of the coupled soil-structure system and spurious non-physical modes of the PML. The latter can be identified and directly disregarded as they cluster below a threshold frequency which depends on the PML stretch function parameterization. The physical modes can be separated into two categories: (a) local modes of the structure which tend to have small to moderate damping and (b) global modes of the generalized structure which generally are heavily damped. A criterion based on the elastic (or kinetic) energy of the system is proposed to sort the modes into these categories. To this end, a weighted characteristic depth y_m of modal elastic (or kinetic) energy is defined:

$$y_m = \frac{\sum_{l=1}^{k} y^l E_m^l}{E_m} \tag{11}$$

where E_m is the total elastic (or kinetic) energy of the m-th mode, E_m^l is the corresponding elastic (or kinetic) energy of the l-th element, y^l is the y coordinate of the center of mass of the l-th element and k is the total number of finite elements in the model. For a structure located almost entirely above ground (figure 2), the m-th mode is sorted as structural if $y_m \geq 0$ and as generalized structure mode if otherwise.

3. Case study

The sensitivity of the modal characteristics of a two dimensional reinforced concrete frame structure with respect to the subsoil properties is investigated. The structure is founded on two partially embedded strip foundations with a width of $b_{\mathrm{s}} = 1$ m and plane strain conditions are assumed. The soil is modeled with quadratic elements with an element size of 0.25 m corresponding to 10 finite elements per shear wavelength at a frequency of 40 Hz for the lowest soil shear wave velocity considered in this work (table 2). The PML are parameterized such that any spurious non-physical modes to appear well below 1Hz. The structure is modeled with frame elements where the material properties have been modified as follows to take into account the plane strain conditions:

$$E_{\mathrm{eff}} = \frac{E_{\mathrm{b}}}{1-\nu_{\mathrm{b}}^2} \quad \text{and} \quad \nu_{\mathrm{eff}} = \frac{\nu_{\mathrm{b}}}{1-\nu_{\mathrm{b}}} \tag{12}$$

Figure 3 shows the FE-PML model. Both the RC structure and the soil are assumed undamped. Tables 1 and 2 summarize the parameters of the model. The first four modes of the structure fixed at its base are shown in figure 4. These are two lateral and two vertical modes of the floors. As the structure is modeled without any damping, all the modal damping ratios are zero and the mode shapes are real.

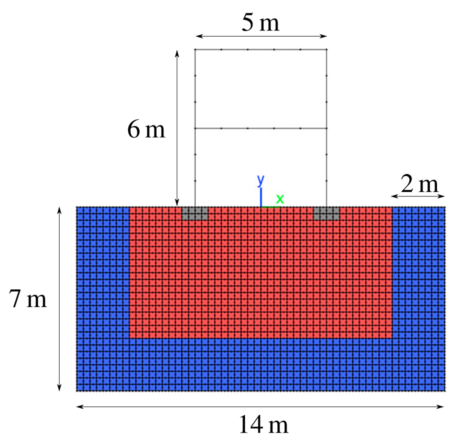

Fig. 3. FE-PML model.

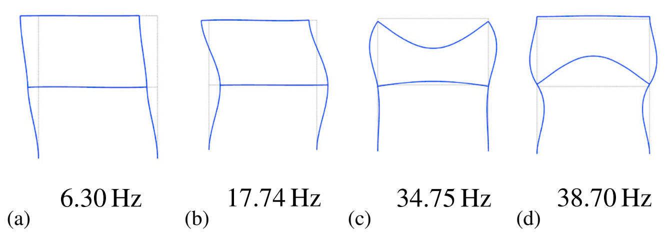

Fig. 4. (a) First, (b) second, (c) third and (d) fourth fixed base modes of the structure.

Table 1. R/C building properties.

Columns [m × m]	Beams [m × m]	E_b [GPa]	ν_b [−]	ρ_b [kg/m^3]
0.30 × 1.00	0.60 × 1.00	30	0.30	2500

Table 2. Soil properties.

h [m]	C_s [m/s]	C_p [m/s]	ρ [kg/m^3]
∞	$\{10^2, ..., 10^3, ..., 10^4\}$	$2\,C_s$	1800

The significance of dynamic soil-structure interaction effects strongly depends on the relative stiffness between the soil and the structure [1]. Based on the previously outlined FE-PML methodology, a parametric study for the computation of the the modal characteristics of the frame structure with respect to the shear wave velocity C_s of the soil is performed. Figure 5 shows the modal characteristics of the structure as a function of the shear wave velocity C_s of the soil. The modal characteristics are presented in terms of ratios of the coupled soil-structure eigenfrequencies ω to the fixed base structure eigenfrequencies ω_s, modal radiation damping ratios ξ, MAC values between the coupled soil-structure mode shapes $\boldsymbol{\psi}_m$ and the fixed base structure mode shapes $\boldsymbol{\phi}_m$, and modal collinearity factors (MCF). The modal assurance criterion (MAC) indicates the correspondence between two mode shapes and is defined as:

$$\mathrm{MAC}_m(\boldsymbol{L}\boldsymbol{\psi}_m, \boldsymbol{\phi}_m) = \frac{|\,(\boldsymbol{L}\boldsymbol{\psi}_m)^{\mathrm{H}}\,\boldsymbol{\phi}_m|^2}{||\boldsymbol{L}\boldsymbol{\psi}_m||_2^2||\boldsymbol{\phi}_m||_2^2} \tag{13}$$

where $\boldsymbol{L} \in \mathbb{R}^{\mathrm{n_b}\times\mathrm{n}}$ is a matrix selecting the n_b degrees of freedom of the superstructure from the n degrees of freedom of the FE-PML model and the superscript H denotes the Hermitian of a vector. MAC values close to 1 indicate better correspondence of the compared mode shapes. When the MAC values are lower than 0.75, the correspondence between the coupled soil-structure and the fixed base structure mode shapes $\boldsymbol{\psi}_m$ and $\boldsymbol{\phi}_m$ is rather weak for the corresponding shear wave velocities C_s of the soil.

Dynamic soil-structure interaction effects manifest as lower eigenfrequencies for the coupled soil-structure system, increase in modal damping ratios and occurrence of complex valued mode shapes. Since the structure and the soil are modeled without any inherent damping mechanisms, the increase in modal damping ratios is solely attributed to radiation damping as energy loss throughout the soil. To this matter, it is noted that in three-dimensional problems the radiation damping is expected to be even more significant. All the modes of the structure are affected, with the influence getting more profound for soft soils. The fundamental eigenfrequency and modal damping ratio of the coupled soil-structure system are highly sensitive to the stiffness of the soil, whereas the corresponding mode shape is not (figure 5a). The vertical modes of the floors (figures 5c and 5d) exhibit more intensive interaction with the soil than the lateral modes of the structure (figures 5a and 5b). This happens because for a wide range of shear wave velocities C_s the soil gets particularly activated by the vertical modes of the coupled soil-structure system. Unlike the fundamental mode of the system, the second vertical mode is mostly sensitive in terms of its mode shape rather than its eigenfrequency or modal damping ratio. Finally, the number of relevant modes in a given frequency range can be higher when considering dynamic soil-structure interaction effects. For example, when the shear wave velocity C_s of the soil is less than 200 m/s, the coupled soil-structure system exhibits two modes that can be qualified as lateral and compared to the second lateral mode of the fixed base structure (figure 5b).

4. Conclusions

In this paper, a FE-PML model is used for the computation of the modal characteristics of a frame structure considering dynamic soil-structure interaction effects. The modal characteristics of the structure are computed for a wide range of shear wave velocities of the soil. The investigation shows that dynamic soil-structure interaction effects influence all modes of the structure, and in particular the vertical modes where soil-structure interaction plays a larger role even in the case of stiff soils. Dynamic soil-structure interaction generally leads to lower eigenfrequencies for the coupled soil-structure system, increase in modal damping ratios and occurrence of complex valued mode shapes. By considering this interaction, modes of the coupled soil-structure system can be observed which would be otherwise intractable.

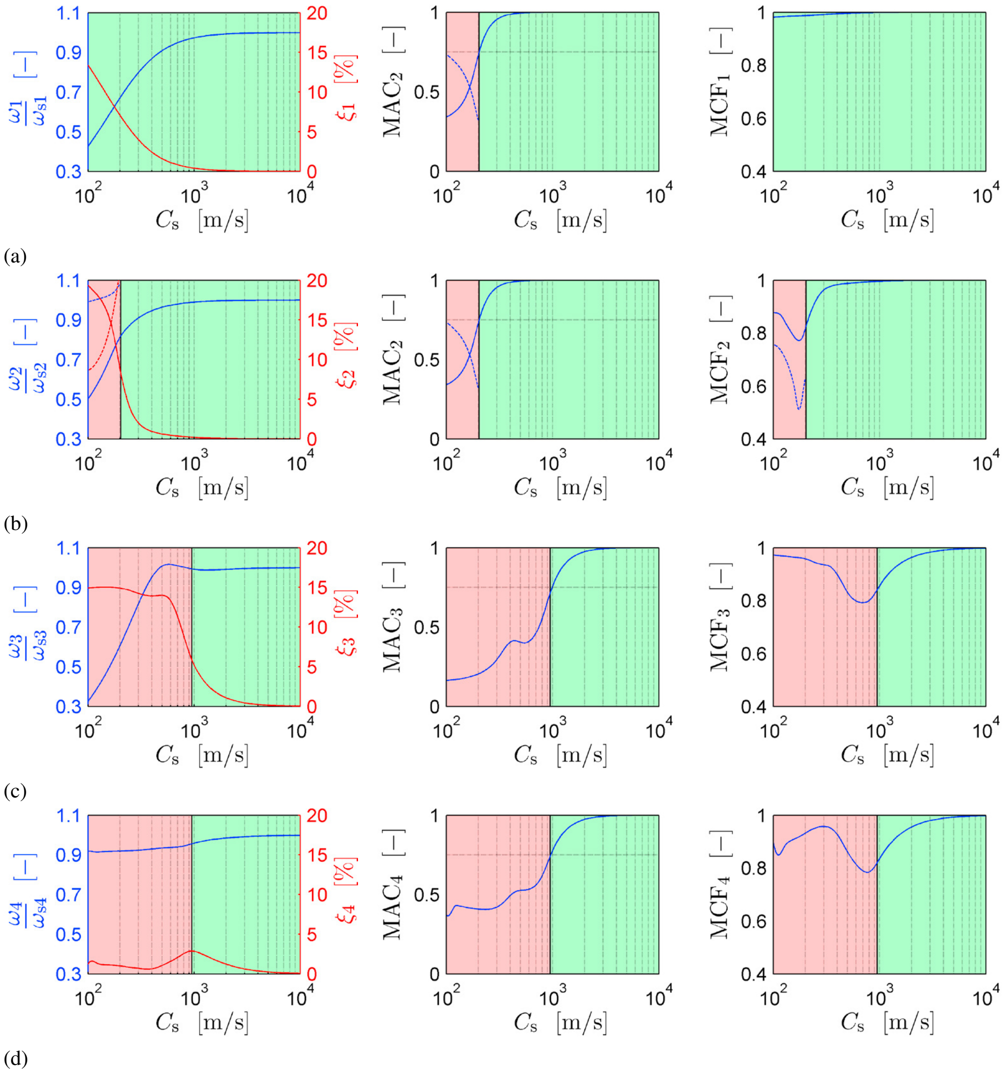

Fig. 5. Modal characteristics in terms of eigenfrequency ratios ω_i/ω_{si}, modal damping ratios ξ_i and MCF_i of the coupled soil-structure system for a range of shear wave velocities C_s of the soil. The parts of the graphs corresponding to MAC < 0.75 are indicated with red background. (a) First, (b) second, (c) third and (d) fourth structural mode of the coupled soil-structure system.

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